RuleML - Semantic Profile for the First Order Deontic Alethic Logic

Adrian Paschke

RuleML Inc., Canada
`paschke AT inf.fu-berlin.de`

Abstract. This is the description of the Semantic Profile for the **First Order Deontic Alethic Logic**. This profile can be used, e.g. in the **Knowledge Representation (KR) dialect** of **Reaction RuleML**.

1 Introduction

This document describes the Semantic Profile for the **First Order Deontic Alethic Logic** (FODAL) [4].

2 Signature

Definition 1. *(**Alphabet**) The alphabet Σ consists of the following class of symbols:*

- *A signature $S = \langle \overline{P}, \overline{F}, arity, \overline{c} \rangle$, with*
  - *$\overline{P}$ an infinite set of predicate symbols $\langle P_1, .., P_n \rangle$.*
  - *$\overline{F}$ a infinite set of function symbols $\langle F_1, .., F_m \rangle$*
  - *For each P_i respectively each F_j, $arity(P_i)$ resp. $arity(F_j)$ is a non-zero natural number denoting the arity of P_i resp. F_i.*
  - *$\overline{c} = \langle c1, .., c_o \rangle$ is a finite or infinite sequence of constant symbols.*
- *A collection of variables V which will be denoted by identifiers starting with a capital letter like U,V,X*
- *Logical connectives / operators: $\neg$. (negation), $\wedge$ (conjunction), $\vee$ (disjunction), $\rightarrow$ (implication), $\leftrightarrow$ (iff) and $\equiv$ (equivalent).*
- *Modal connectives / operators: $\Box$ (alethic necessity), $\diamond$ (alethic possibility),* O *(deontic obligation),* P *(deontic permission),* F *(deontic forbidden).*
- *Quantifier: $\forall$ (forall), $\exists$ (exists).*
- *Parentheses symbols: "(", ")".*

A formula ϕ is defined as in FOL with the extension of a set of modal formulas ϕ^{Mod} ($\Box\phi$, $\diamond\phi$, $O\phi$, $P\phi$, $F\phi$) with the additional modal operators ($\diamond$, P, F) definable in terms of the others:

- $\diamond\phi$ (possibly ϕ) $\equiv \neg\Box\neg\phi$ (not necessarily not ϕ")
- $P\phi$ (permitted ϕ) $\equiv \neg O \neg\phi$ (not obligatory that not ϕ)
- $F\phi$ (forbidden ϕ) $\equiv O\neg\phi$ (obligatory that not ϕ)

2.1 Semantics

The semantics is defined by a two layered Kripke semantics with augmented bimodal frames consisting of two accessibility relations, R_O and $R_\Box$ between possible worlds.

Definition 2. ***(Augmented frame)*** *A varying domain augmented bimodal frame $A = \langle W, R_\mathrm{O}, R_\Box, d\rangle$ consists of a non-empty set, W, whose members are possible worlds, two binary accessibility relations, R_O and $R_\Box$, that hold (or not) between the possible worlds of W, a domain function d mapping possible worlds w to a non-empty set P such that if $d(w, P)$, then P is true at w.*

As in S4 the alethic accessibility relation is reflexive and transitive [1] and the deontic accessibility relation is serial as in KD [3]. The FODAL semantics additionally defines the bi-modal FODAL frame with the modal formula for the interaction between alethic and deontic logic.

– $\Box\phi \rightarrow O\phi$ (Everything which is necessary is also obligatory)

Definition 3. ***(Interpretation and model)*** *An interpretation I in an augmented frame A is an interpretation function which assigns to each possible world w and each predicate symbol p some n-ary relation to the domain $D(w)$ of that world. A model M is an interpretation of an augmented FODAL frame A, if A is true wrt to I.*

The satisfiability relation between FODAL models and formulae is then defined in the usual way: ϕ is a FODAL formula and σ is an assignment to the interpretation I, then the relation $I \models \phi[\sigma]$ means that ϕ is true in I when there is a substitute for each free variable V of ϕ with the value of $\sigma(V)$. We omit the definition of the inductive requirements of "$\models$" here and refer to [4]. Accordingly, a formula ϕ is satisfied by an interpretation I ($I \models \phi$) iff $I \models_\sigma \phi$ for all variable assignments σ.

3 Axioms

Following [2] the formalization is given as an axiomatic system in the typical way for a normal modal logic. The FODAL axiomatization is obtained by combining the axiom systems of S4 and KD and extending it with the additional axioms defining the relations between alethic and deontic modalities.

Definition 4. ***(Axioms)***

– *All S4 and KD tautologies and axioms*
– *All instances of the Kripke schema:* $\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$ *and* $\mathrm{O}(A \rightarrow B) \rightarrow (\mathrm{O}A \rightarrow \mathrm{O}B)$
– *(Vacuous $\forall$)* $\forall x\phi \equiv \phi$ *with x not being free in ϕ*
– *($\forall$ Distributivity)* $\forall x(\phi \rightarrow \psi) \rightarrow (\forall x\phi \rightarrow \forall x\psi)$
– *($\forall$ Permutation)* $\forall x\forall y\phi \rightarrow \forall y\forall x\phi$

– *($\forall$ Elimination)* $\forall y(\forall x \phi(x) \to \phi(y))$
– *(Necessary O)* $\Box \phi \to \mathrm{O}\phi$

and additionally inference rules

– *(Detachment)* $\frac{\phi \; \phi \to \psi}{\psi}$
– *(Necessiation)* $\frac{\phi}{\Box \phi}$ *and* $\frac{\phi}{(O)\phi}$
– *($\forall$ Generalization)* $\frac{\phi}{\forall x \phi}$

4 Mapping of the Reaction RuleML KR dialect into the FODAL Profile

The translation is defined as the inverse translation function $\tau_{RRML}(\cdot)^{-1}$ from normalized mono-modal FODAL formulas to RRML as follows:

– for each constant c, $\tau^{-1}_{RRML}(c)$ maps it
  - to a data term $< Data >$ in $RRML$ if the constant has an interpretation as a data type in the the XML Schema data types.
  - to an individual term $< Ind >$ in $RRML$ otherwise.

– for each variable v, $\tau^{-1}_{RRML}(v)$ maps it to a variable $< Var >$ in $RRML$.
– for each unary predicate p in $FODAL$, $\tau^{-1}_{RRML}(p)$ maps its only argument term (a constant or a variable) into a term in $RRML$ and assigns the predicate relation p_r as type attribute to the $RRML$ term @$type =$ "p_r".
– for each n-ary predicate p in $FODAL$, $\tau^{-1}_{RRML}(p)$ maps it into an n-ary atom $< Atom >$ in $RRML$ using the predicate relation as relation $< Rel >$ for the atom and each argument term in the $FODAL$ predicate p is mapped into a typed term in the $RRML$ atom, where the type is coming from the previous mapping of a unary predicate which gives the type of the term.
– for each formula R in $FODAL$, $\tau^{-1}_{RRML}(R)$ is defined inductively as follows:
  - $\tau^{-1}_{RRML}(\hat{R})$ maps into a corresponding $RRML$ formula $< formula >$, where $\hat{R}$ is a non-modal first-order logic formula and the $RRML$ formula is its non-modal RRML translation. In particular:
    - if $\hat{R}$ is a conjunction it is mapped into $< And >$.
    - if $\hat{R}$ is a disjunction it is mapped into $< Or >$.
    - if $\hat{R}$ is an implication (or a formula which logically corresponds to an implication) it is mapped into $< Rule >$.
    - if $\hat{R}$ is a universal quantifier or existential quantifier it is mapped into a quantifier $< Forall >$ (might be left implicit if no further constraints are defined on the quantifier) or $< Exists >$, with the declared variables being typed @$type$ with their type (see unary predicate mapping) and additional quantifier constraints defined in the $RRML$ quantifier ("such that" $< formula >$ and guard constraints $< guard >$).
  - $\tau^{-1}_{RRML}(\neg R)$ maps into a $RRML$ negation $< Neg >$ with $\tau^{-1}_{RRML}(R)$ being the corresponding $RRML$ formula which is negated.
  - $\tau^{-1}_{RRML}(R_1 \circ R_2)$, where R_1 and R_2 are FODAL formulas and $\circ \in \{\wedge, \vee, \to, \leftrightarrow\}$ maps into $\tau^{-1}_{RRML}(R_1)\tau^{-1}_{RRML}(\circ)\tau^{-1}_{RRML}(R_2)$, with $\tau^{-1}_{RRML}(\circ) = \{\wedge =< And >, \vee =< Or >, \to =< Rule >, \leftrightarrow =< Equivalent >\}$.

- $\tau^{-1}_{RRML}(\Box\hat{R})$ maps the alethic necessary operator $\Box$ into $< rrml : Operatortype = "rrml : AlethicOperator" iri = "rrml : Necessary" >$ and $\tau^{-1}_{RRML}(\hat{R})$ mapped into its corresponding $RRML$ formula and $\tau^{-1}_{RRML}(\mathbf{O}\hat{R})$ maps the deontic obligation operator $\mathbf{O}$ into $< rrml : Operatortype = "rrml : DeonticOperator" iri = "rrml : Obliged" >$ in $RRML$ and $\tau^{-1}_{RRML}(\hat{R})$ mapped into its corresponding $RRML$ formula. For the other alethic and deontic operators τ^{-1}_{RRML} gives a similar mapping.

5 Profile Usage in Reaction RuleML 1.0

The profile can be defined as intended semantics by the < **Profile** > element in the < **evaluation** > role. The profile type defined in the RuleML vocabulary is:

FirstOrderDeonticAlethicLogic

6 Example

This example shows how to use the profile be defining it as a profile type from the RuleML vocabulary.

```
<evaluation>
  <Profile type="ruleml:FirstOrderDeonticAlethicLogic"/>
</evaluation>
```

References

1. Patrick Blackburn, Maarten de Rijke, and Yde Venema. *Modal Logic*. Number 53 in Cambridge Tracts in Theoretical Computer Science. Cambridge University Press, 2001.
2. Melvin Fitting and Richard L. Mendelsohn. *First-order Modal Logic*. Kluwer Academic Publishers, Norwell, MA, USA, 1999.
3. Paul McNamara. Deontic logic. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Stanford University, fall 2010 edition, 9 2010.
4. Dmitry Solomakhin, Enrico Franconi, and Alessandro Mosca. Logic-based reasoning support for SBVR. In *Proceedings of the 26th Italian Conference on Computational Logic (CILC2011), Pescara, Italy, 31 August 31-2 September, 2011*, pages 311–325, 2011.