RuleML - Semantic Profile for the Reified Classical Situation Calculus

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Abstract. This is the description of the Semantic Profile for the Reified Classical Situation Calculus. This profile can be used, e.g. in the Knowledge Representation (KR) dialect of Reaction RuleML.

1 Introduction

A Semantic Profile\(^1\) in RuleML defines the intended semantics of a RuleML knowledge base as needed e.g. for rule interchange and knowledge imports, updates and querying, as well as semantically safe translations from RuleML into platform specific languages for the execution and reasoning.

This document describes the Semantic Profile for the Reified Classical Situation Calculus. This profile can be used, e.g. in the Knowledge Representation (KR) dialect of Reaction RuleML \(^2\). The formalization is based on Pinto’s axioms \(^2\) which itself originates from Reiter’s formalization that appeared in \(^3\).

2 Signature

The signature is a multi-sorted signature \(\Sigma\) defined as a tuple 
\[
\langle A^T, F^T, S^T, D^T; E, F, S, D, \text{arity, sort, <, do, holds, } S_0, \land, \lor, \neg, \rightarrow, \equiv, \forall, \exists \rangle
\]
with sorts \(A^T\) for action type \((T)\) symbols, sorts \(F^T\) for fluent type symbols, sorts \(S^T\) for situation type symbols, and sorts \(D^T\) for domain objects; \(A\) is the non-empty set of action function symbols of sort \(A^T\), called actions, \(S\) is a non-empty set of situation function symbols of sort \(S^T\), called situations, \(F\) is a non-empty set of fluent function symbols of sort \(F^T\), called fluents, \(D\) is the signature of the domain language which includes domain constant symbols, function symbols, and predicate symbols. The function \(arity(A_i)\), \(arity(F_i)\), \(arity(S_i)\), \(arity(D_i)\) associates a non-zero natural number with each action \(A_i\), fluent \(F_i\), situation \(S_i\) and \(D_i\). The function \(sort\) associates with each n-ary action, fluent or situation function symbol a \(n+1\)-tuple of sorts. That is, if \(expr\) is an action, fluent, or situation function of arity \(n\), then \(sort(expr)\) is a \(n+1\)-tuple

\(^1\) http://wiki.ruleml.org/index.php/Glossary_of_Reaction_RuleML_1.0#gloss-Profile
\(^2\) http://reaction.ruleml.org/ and http://wiki.ruleml.org/index.php/Specification_of_Reaction_RuleML_1.0#Quick_Links
of sorts \(\text{sort}(expr) = (T_1, \ldots, T_n, T_{n+1})\) where \((T_1, \ldots, T_n)\) defines the sorts (types) of the domain of \(expr\) and \(T_{n+1}\) defines the sort of the range of \(expr\), where each \(T_i\) is some sort in \(A^T, F^T,\) or \(S^T\). Similarly, \(\text{sort}(D)\) gives the sorts (types) \(D^T\) of \(D\). Furthermore, the signature includes the relation \(\langle\subseteq S^T \times S^T\), which defines a \textit{history} ordering of situations, the function \textbf{do} : \(A^T \times S^T \rightarrow S^T\), the predicate \textbf{holds} \(\subseteq F^T \times S^T\) the relation \textbf{Poss} \(\subseteq F^T \times S^T\), and the special constant \(S_0\) of sort \(S^T\) which is the initial situation. \(\land, \lor, \rightarrow, \neg \equiv\) are the standard operators for conjunction, disjunction, implication, equivalence (iff), and negation and \(\forall, \exists\) are standard universal and existential quantifiers.

In the following, the smaller case letters \(a\) (actions), \(s\) (situations), \(f\) (fluents), \(d\) (domain) are used for variables and upper case \(A, S, F, D\) are used for constants. \(\varphi\) is a predicate variable over functions of arity 1. Unless stated with an explicit quantifier, free variables are always assumed to be universally quantified. \(s_1 \leq s_2\) is an abbreviation for \(s_1 < s_2\lor s_1 = s_2\). Another used abbreviation is \(\text{do}(a_1, \ldots, a_n, s)\) for \(\text{do}(a_n, \text{do}(\ldots, \text{do}(a_1, s)\ldots))\).

### 3 Axioms

\[
(\forall \varphi) \cdot [\varphi(S_0) \land (\forall s, a)(\varphi(s) \rightarrow \varphi(\text{do}(a, s)))] \rightarrow (\forall s)\varphi(s), \tag{1}
\]

\[
(\forall a_1, a_2, s_1, s_2). \text{do}(a_1, s_1) = \text{do}(a_2, s_2) \rightarrow a_1 = a_2, \tag{2}
\]

\[
(\forall s_1, s_2, a). s_1 < \text{do}(a, s_2) \equiv s_1 \leq s_2, \tag{3}
\]

\[
(\forall s_1, s_2). s_1 < s_2 \rightarrow \neg s_1 < s_2, \tag{4}
\]

\[
(\forall s_1, s_2). s_1 < s_2 \rightarrow \neg s_1 < s_2, \tag{5}
\]

\[
s_1 < s_2 \equiv (\exists s_2). (s_2 = \text{do}(a_1, a_2, \ldots, a_k, s_1)), \tag{6}
\]

Note: \(s_1 < s_2\) is true iff there exists a sequence of actions \(a_1, a_2, \ldots, a_k\) is weaker than Reiter’s \(\leq\) in which every action additionally must be possible:

\[
\neg s \leq S_0, \tag{7}
\]

\[
s_1 \leq \text{do}(a, s_2) \equiv \text{Poss}(a, s_2) \land s_1 < s_2. \tag{8}
\]

### 4 Propositions

\[
(\forall s)S_0 \leq s, \tag{9}
\]

\[
(\forall s)\neg s \leq s, \tag{10}
\]

\[
(\forall s)\neg s \leq S_0, \tag{11}
\]

\[
(\forall s). s \neq S_0 \rightarrow (\exists a, s_2) s = \text{do}(a, s_2), \tag{12}
\]

\[
(\forall s_1, s_2, s_3). s_1 < s_2 \land s_2 < s_3 \rightarrow s_1 < s_3, \tag{13}
\]

\[
(\forall a, s). s < \text{do}(a, s), \tag{14}
\]
(∀a₁, a₂, s₁, s₂).do(a₁, s₁) < do(a₂, s₂) \rightarrow s₁ < s₂, \quad (15)
(∀a₁, s₁).do(a₁) ≤ s₂ \rightarrow s₁ < s₂, \quad (16)
(∀a₁, s₁).do(a₁) = do(a, s₂) \rightarrow s₁ = s₂, \quad (17)
(∀a₁, s₁).¬(s₁ < s₂ < do(a, s₁)), \quad (18)
(∀s₁, s₂).s₁ < s₂ \rightarrow s₁ < s₂, \quad (19)
(∀f₁, s₁, s₂).s₁ < s₂ \land holds(f₁, s₁) \land \neg holds(f, s₂) \rightarrow
(∃s, a)(s₁ < do(a, s) \leq s₂) \land \neg[holds(f, do(a, s)) \equiv holds(f, s)]. \quad (20)

5 Mapping of the Reaction RuleML KR dialect into the Reified Situation Calculus Profile

Given an Reaction RuleML 1.0 document .rrml modelled in the Knowledge Representation (KR) dialect of Reaction RuleML (KR Reaction RuleML 1.0) the following translation \(\tau(\cdot)\) from elements of RR1 to sentences of the situation calculus profile logic is defined:

- for each \(<\text{Fluent}>\) expression function with \(n\) arguments in .rrml, \(\tau(<\text{Fluent}>)\) is a fluent \(F_i \in \mathcal{F}\) of arity \(n\) with the optional attribute value of @type (or the default sort ruleml : Fluent) denoting the sort of \(F_i\) in \(F_T\),
- for each \(<\text{Action}>\) expression function with \(n\) arguments in .rrml, \(\tau(<\text{Action}>)\) is an action \(A_i \in \mathcal{A}\) of arity \(n\) with the optional attribute value of @type (or the default sort ruleml : SimpleAction) denoting the sort of \(A_i\) in \(A_T\),
- for each \(<\text{Situation}>\) expression function with \(n\) arguments in .rrml, \(\tau(<\text{Situation}>)\) is a situation \(S_i \in \mathcal{S}\) of arity \(n\) with the optional attribute value of @type (or the default sort ruleml : History) denoting the sort of \(S_i\) in \(S_T\),
- for each \(<\text{Do}>\) function in .rrml with first argument an \(<\text{Action}>\) and second argument a \(<\text{Situation}>\), \(\tau(<\text{Do}>)\) is a 2-ary function \(do\),
- for each predicate \(<\text{Atom}>\) in .rrml with relation name \(<\text{Rel}>\) poss \(<\text{/Rel}>\) and first argument an \(<\text{Action}>\) and second argument a \(<\text{Situation}>\), \(\tau(<\text{Atom}>)\) is a 2-ary predicate \(Poss\),
- for each predicate \(<\text{Holds}>\) in .rrml with first argument an \(<\text{Fluent}>\) and second argument a \(<\text{Situation}>\), \(\tau(<\text{Holds}>)\) is a 2-ary predicate \(holds\),
- for each individual \(<\text{Ind}>\) \(s0 < <\text{/Ind}>\) in .rrml, \(\tau(<\text{Ind}> s0 < <\text{/Ind}>)\) is the situation constant \(S_0\),
- for all other (Reaction) RuleML elements \(t\) and \(r\) in .rrml, \(\tau(t)\) and \(\tau(r)\) is defined inductively as follows:
  - \(\tau(t) = \phi_t\), where \(t\) is a RuleML term and \(\phi_t\) is its first-order logic translation into a logic term (constans, functions, variables),
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• \( \tau(r) = \phi_r \), where \( r \) is a RuleML formula and \( \phi_r \) is its first-order logic translation into a logic formula,
• \( \tau(\neg r) = \neg \phi_r \),
• \( \tau(r_1 \circ r_2) = \tau(r_1) \circ \tau(r_2) \), \( \circ \in \land, \lor, \neg, \rightarrow \), where \( r_1 \) and \( r_2 \) are formulas, with \( \tau(\cdot) \) maintaining the same arity and quantification and the optional value of the attribute \@type \) (or a default sort from the RuleML metamodel) denoting the sort of \( \cdot \) in \( D^T \).

6 Profile Usage in Reaction RuleML 1.0

The profile can be defined as intended semantics by the \(<\text{Profile}>\) element in the \(<\text{evaluation}>\) role.

– the \@type attribute defines the name/type of the used semantics profile
– the \@iri attribute points to IRI of the semantics profile

The profile IRI is:

http://reaction.ruleml.org/1.0/profiles/ReifiedClassicalSituationCalculusProfile.rrml

The profile type defined in the RuleML vocabulary is:

ReifiedClassicalSituationCalculus

This profile specifies the semantics for the reified situation calculus in terms of a RuleML formalization of the axioms and propositions given in the section 3 and 4. It can be imported by XML Inclusions (XInclude) or the Reaction RuleML \(<\text{Consult}>\) into a KR Reaction RuleML Reified Situation Calculus program.

7 Example

This example shows how to use the profile be defining it as a profile type from the RuleML vocabulary.

```xml
<evaluation>
  <Profile type="ruleml:ReifiedClassicalSituationCalculus"/>
</evaluation>
```

This example shows how to use the profile be referencing it’s IRI.

```xml
<evaluation>
  <Profile iri="http://reaction.ruleml.org/1.0/profiles/ReifiedClassicalSituationCalculusProfile.rrml"/>
</evaluation>
```

This example show how to include the profile with all its axioms and rules into a Reaction RuleML document.

```xml
<!-- include the axioms from profile "SituationCalculusProfile.rrml" -->
<xinclude href="http://reaction.ruleml.org/1.0/profiles/ReifiedClassicalSituationCalculusProfile.rrml" xpointer="/RuleML/*"/>

<!-- use the included "ruleml:ReifiedClassicalSituationCalculus" semantic profile for interpretation -->
<evaluation>
  <Profile keyref="ruleml:ReifiedClassicalSituationCalculus" direction="backward"/>
</evaluation>
```
Further examples can be found in the Reaction RuleML 1.0 examples directory:
http://reaction.ruleml.org/1.0/exa/kr/

8 Acknowledgement

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References